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Probability Distribution Function of the Inter-Pulse Intervals of Single Motor Unit Action Potentials during Isometric Contractions

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A train of motor unit action potentials can be represented by a pulse random process which is illustrated in figure 1. Consider an impulse random process $\delta(t-t_j)$. The impulses are located at time t_j, where t_j is a random variable. If the impulse process is passed through a system with an impulse response h(t), a pulse random process u(t) is formed. If h(t) is the equation of a motor unit action potential, the random process u(t) represents a motor unit action potential train which can be expressed as

$$u(t)=\sum_{j=1}^{n} h(t-t_j) \qquad \text{where} \quad t_j=\sum_{i=1}^{j} x_i$$

for i, j = 1, 2, 3,..., n. The random variable x represents the inter-pulse interval. If the probability distribution function of x were known, it would be possible to construct pulse random processes of the same class as u(t) for any function h(t).

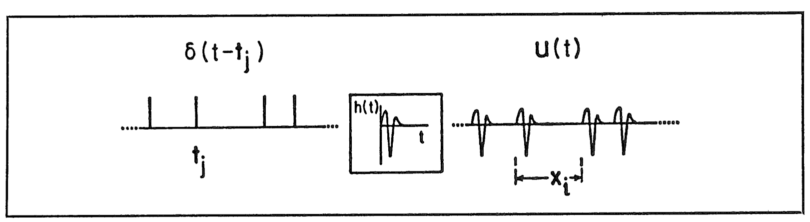

Fig. 1. Pulse random process which can represent a motor unit action potential train.

This article is concerned with the probability distribution function of the inter-pulse intervals for the complete force range and time duration of a constant force isometric muscle contraction.

Experiment

A specially designed quadri-filar needle electrode [De Luca and Forrest, 1972a] was used to record motor unit action potentials from the middle fibers of the deltoid muscle in four male subjects while performing isometric abduction in the coronal plane. With this electrode it was possible to record and identify individual motor unit action potentials during maximal muscle contractions. Each subject was asked to sustain a contraction at a pre-set force level until he could no longer elicit the required force. Between successive contractions, each subject had a minimum rest period of two hours. Recordings of motor unit action potential trains were obtained for various force levels ranging from 16 to 120 kg. This force corresponds to the force output of the middle fibers of the deltoid. A special technique was employed to obtain these values from the monitored force [De Luca and Forrest, 1972b]. The myoelectric signal was recorded on a 35-mm film. The inter-pulse intervals of 70 motor unit action potential trains were measured by visual inspection. Two persons independently interpreted the records, thus reducing the subjectivity of the measurements.

Properties of the Inter-Pulse Interval Histograms

Histograms of the inter-pulse interval for each complete motor unit train were plotted. A typical histogram can be seen in figure 2. In each case, the mean, SD, skewness, minimum and maximum values of the inter-pulse interval were calculated. In 52 histograms, the mean was larger than the SD; in 3 the mean and the SD were approximately equal; and in the remaining 15 the SD was slightly larger than the mean. The envelope of all the histograms clearly demonstrated a positive skewness. These observations agree with those of Lippold *et al.* [1960] and disagree with the observations of Clamann [1968] and Leifer [1969]. Lippold *et al.* recorded from the triceps muscle; both Clamann and Leifer used the biceps brachii muscle.

Each motor unit train was divided into 10 equal time-sections. The histograms of each section of all the motor unit trains were plotted; 700 histograms were formed. Figure 3 shows the histograms of a typical sectioned motor unit train. The following observations can be made: (a) the positive skewness persists in all the sections, and (b) the mean and SD increase with time.

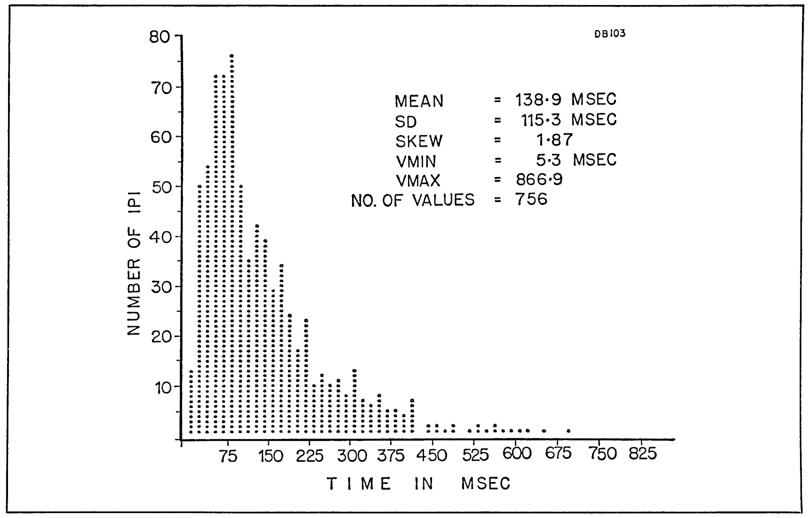

Fig. 2. Histogram of the inter-pulse intervals (IPI) of a motor unit action potential train which was recorded during an isometric contraction; the contraction was sustained until the pre-set constant force could no longer be maintained. The force from the monitored muscle was 25.9 kg.

To elucidate these observations, the mean of each motor unit action potential train was plotted against time. The plot shown in figure 4 was obtained by calculating the average of the inter-pulse interval every 5 sec. It can be seen that the mean inter-pulse interval increases with time. The mean value was also plotted against the SD (fig. 5). Again, the plot was obtained by calculating the mean value and SD for every 5-sec interval. A polynomial least-square regression was performed on each motor unit action potential train. Figure 5 contains curves for 16 motor unit action potential trains obtained from 1 subject; the contractions varied from low to maximum force. It is apparent that the mean and SD remain approximately constant with respect to each other.

A linear least-square regression was performed on the values of the SD against the mean values of the inter-pulse intervals for every 5-sec interval in all 70 motor unit action potential trains. The coefficient of regression (the slope) was found to be 0.69 and the correlation coefficient was 0.83. The slope, i.e. the ratio of SD to the mean, is called the coefficient of variation.

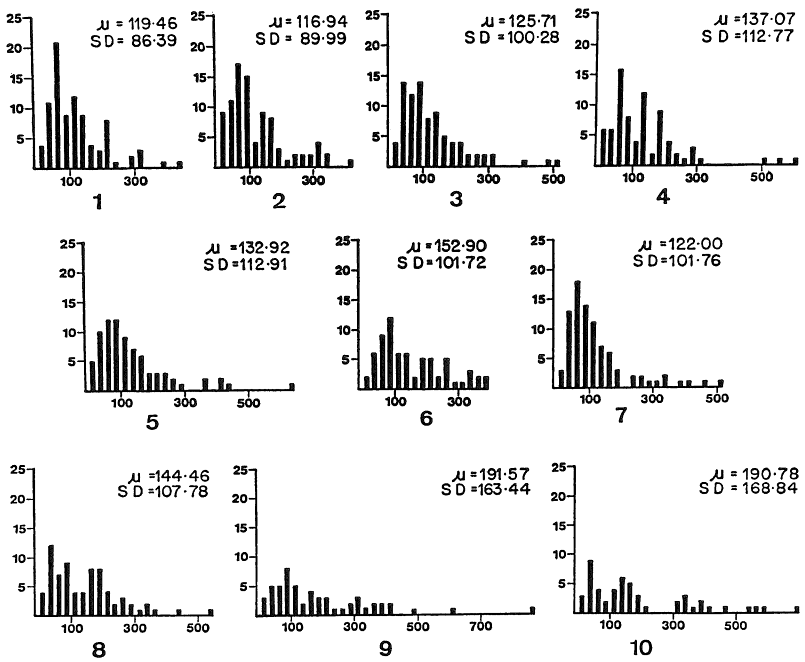

Fig. 3. Histograms of 10 equal and consecutive time-sections of a motor unit action potential train that was recorded during an isometric contraction; the contraction was sustained until the pre-set constant force could no longer be maintained. The abscissa is scaled in msec.

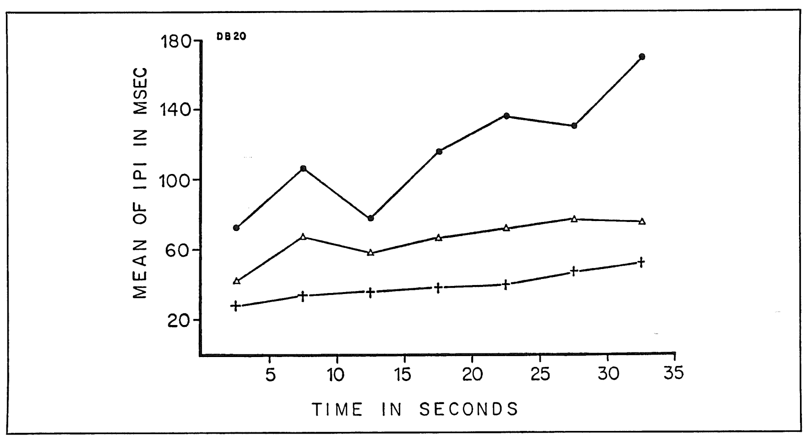

Fig. 4. Time dependence of the mean value of the inter-pulse intervals (IPI); the force output of the recorded muscle was 45.3 kg.

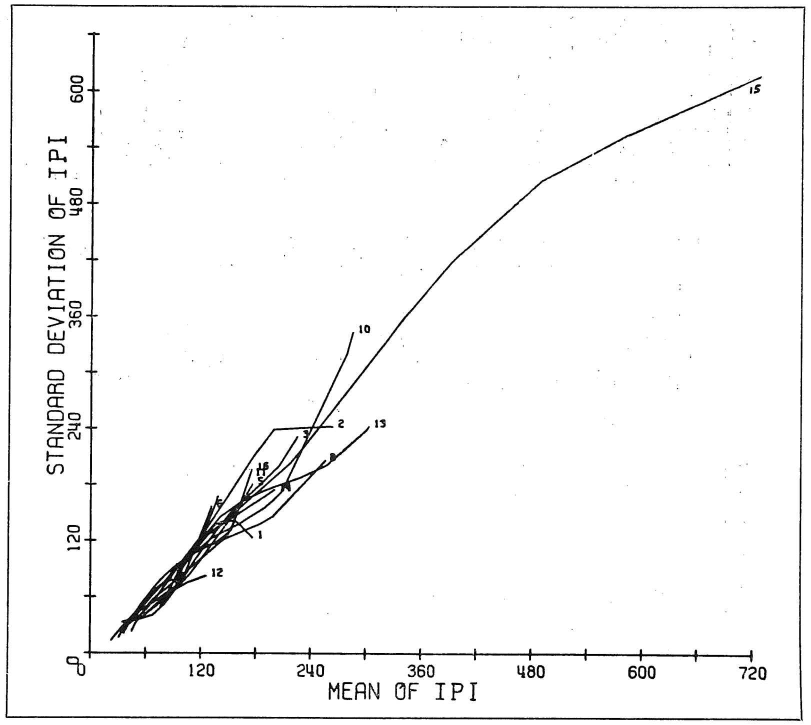

Fig. 5. Relationship of the mean and SD of the inter-pulse intervals (IPI) of 16 motor unit action potential trains recorded from 1 subject; contraction force varied from minimum to maximum. The abscissa and the ordinate are scaled in msec.

Probability Distribution Function of Inter-Pulse Intervals

The histograms mentioned above were fitted with 3 probability distribution functions which have the required properties. They were the Gamma, Lognormal and Weibull probability distribution functions. Due to space limitation, only the Weibull probability distribution function will be discussed in this article. The function is represented by the following equation

$$pX(x)=\frac{\kappa}{\beta}\left[\frac{x-\alpha}{\beta}\right]^{\kappa-1}\exp\left[-\left(\frac{x-\alpha}{\beta}\right)^{\kappa}\right]$$

where x = the inter-pulse interval, κ = shape parameter, β = scale parameter,

a = minimum value of the inter-pulse interval. The mean values, SD and coefficient of variation can be expressed as

$$\text{mean} = \beta\,\Gamma\left(1+\frac{1}{\kappa}\right) \qquad \text{SD} = \beta\left\{\Gamma\left(1+\frac{2}{\kappa}\right) - \left[\Gamma\left(1+\frac{1}{\kappa}\right)\right]^2\right\}^{1/2}$$

$$\text{coefficient of variation} = \left\{\frac{\Gamma\left(1+\frac{2}{\kappa}\right)}{\left[\Gamma\left(1+\frac{1}{\kappa}\right)\right]^2} - 1\right\}^{1/2}$$

where Γ = the gamma function.

The required properties of the inter-pulse interval histograms are satisfied if $\kappa > 1$.

A computer program was written to obtain the values of κ and β that 'best' describe the probability distribution function of the inter-pulse intervals. The 'best' values of κ and β were calculated by the Maximum Likelihood method and the goodness-of-fit of the probability distribution function was measured by the Kolmogorov-Smirnov test.

Out of the 70 motor unit action potential trains tested, the Weibull probability distribution function provided a better fit than either the Lognormal or Gamma probability distribution functions for 38 motor unit action potential trains. Due to the time dependency of the inter-pulse intervals, a better appreciation of the goodness-of-fit of the Weibull probability distribution function can be obtained by applying the test on sections of the motor unit action potential trains. Each motor unit action potential train was divided into groups of not less than 150 consecutive motor unit action potentials; 224 such groups were formed. Table I contains 10 probability levels of the Kolmogorov-Smirnov statistics and the corresponding number and percentage of sections of motor unit action potential trains in each probability level.

The results in table I show a clear delineation between the goodness-of-fit of the Weibull probability distribution function and the other two probability distribution functions. A χ^2 test with 9 degrees of freedom was performed to measure the uniformity of the results. The Gamma and the Lognormal probability distribution functions still provide very bad fits with significance levels of $p<0.00001$. The results for the Weibull probability distribution function have a significance level of $p=0.41$. This is strong evidence that the Weibull probability distribution function provides a good fit for the interpulse intervals of the motor unit action potential trains recorded at all levels of constant force isometric contraction.

Table I. Number and percentage of motor-unit action-potential train sections at 10 levels of probability of acceptance

Kolmogorov-Smirnov probability	level	Gamma		Lognormal		Weibull	
		number	percent	number	percent	number	percent
0	0.1	98	44	91	41	25	11
0.1	0.2	37	17	30	13	19	8.5
0.2	0.3	26	12	17	7.6	21	9.4
0.3	0.4	23	10	11	4.9	30	13
0.4	0.5	8	3.6	12	5.4	26	12
0.5	0.6	6	2.7	10	4.5	28	13
0.6	0.7	12	5.4	12	5.4	18	8.0
0.7	0.8	4	1.8	10	4.5	23	10
0.8	0.9	6	2.7	13	5.8	19	8.5
0.9	1.0	4	1.8	18	8.0	15	6.7

Total number of sections = 224.

Trends in the Parameters of the Weibull Probability Distribution Function

The parameter, α, was found to be uncorrelated with either time or force. The average value of α for all the complete motor unit action potential trains was 3.79 ± 2.82 msec.

A multiple linear least-square regression was performed on the values of κ and the natural logarithm of β ($\ln \beta$) of the sectioned motor unit action potential trains with respect to time and constant force. Motor unit action potential trains recorded from all 4 subjects were used. Time was expressed as a fraction of the total time duration of the motor unit action potential train, and the constant force as a fraction of the maximum force output of the middle fibers of the deltoid muscle. Table II lists the results of the multiple linear least-square regression. The parameter κ is unitless; β has units of msec.

The mean and standard error of the time and constant-force coefficients were calculated. A t-test was performed to establish the significance level of the time and constant-force coefficients; in all cases, the t-test indicated highly significant results. To further elucidate the validity of the linear relationship, the residuals of κ and $\ln \beta$ were separately plotted against their corresponding (a) predicted value, (b) normalized time, and (c) normalized

Table II. Time and force dependence of the Weibull probability distribution function parameters

Parameter	Constant	Regression coefficients					
		time coefficient			constant-force coefficient		
		mean	standard error	p value of t-test	mean	standard error	p value of t-test
κ	1.16	−0.19	0.03	0.000001	0.18	0.05	0.0001
ln (β)	4.60	0.67	0.12	0.000001	−1.16	0.17	0.000001

constant-force. In all 6 plots, the plotted values were randomly dispersed indicating that a linear relationship is as good as any other relationship. The residual of κ or ln β is defined as the difference between the actual value of κ or ln β and their corresponding estimated value from the linear regression.

The average time and constant-force dependence of κ and β can be expressed by the following equations:

$$\kappa(\tau,f) = 1.16 - 0.19\tau + 0.18f$$

$$\beta(\tau,f) = \exp(4.60 + 0.67\tau - 1.16f) \text{ msec}$$

for $0 < \tau < 1$ and $0 < f < 1$.

Properties of the Weibull Probability Distribution

The equation for the inter-pulse intervals can be obtained by taking the cumulative distribution function of the Weibull probability distribution function, which can be rearranged and expressed as

$$x = -\beta(\tau,f)\,[\ln D]^{\frac{1}{\kappa(\tau,f)}} + a$$

where x = inter-pulse interval, D = random variable with uniformly distributed values between 0 and 1, τ = time expressed as a fraction of the total time duration of the motor unit action potential train, f = constant force expressed as a fraction of the force of the maximum voluntary isometric contraction, $\beta(\tau,f)$, $\kappa(\tau,f)$ = time- and force-dependent parameters of the Weibull probability distribution function, a = minimum value of the inter-pulse intervals.

The Survivor function and the Hazard function can also be obtained. The Survivor function

$$\psi(y,\tau,f) = \exp\left[-\left(\frac{y-a}{\beta(\tau,f)}\right)^{\kappa(\tau,f)}\right]$$

gives the probability that a motor unit has not fired up to time y measured from the time of the previous firing. The Hazard function

$$\Phi(y,\tau,f) = \frac{\kappa(\tau,f)}{\beta(\tau,f)}\left(\frac{y-a}{\beta(\tau,f)}\right)^{\kappa(\tau,f)-1}$$

gives the probability of an immediate firing when no firing has occurred for y time. For $\kappa(\tau,f)>1$, there is positive aging with $\Phi(y,\tau,f)$ varying from zero to infinity as y increases.

Discussion

The Weibull probability distribution function with time- and force-dependent parameters can be used to describe the inter-pulse intervals of a motor unit action potential train for the complete force range and time duration of constant force isometric contractions in the middle fibers of the deltoid muscle. Subsequently, it is possible to obtain an expression which describes the random duration of the inter-pulse intervals. Such an equation can be used to generate a continuous random variable having the same properties as the inter-pulse intervals recorded in skeletal muscles during constant force isometric contractions. This is an important first step towards a mathematical synthesis of the myoelectric signal.

Additional information concerning the process that controls the firing of a motor unit is disclosed by the Survivor function and the Hazard function of the Weibull probability distribution function.

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