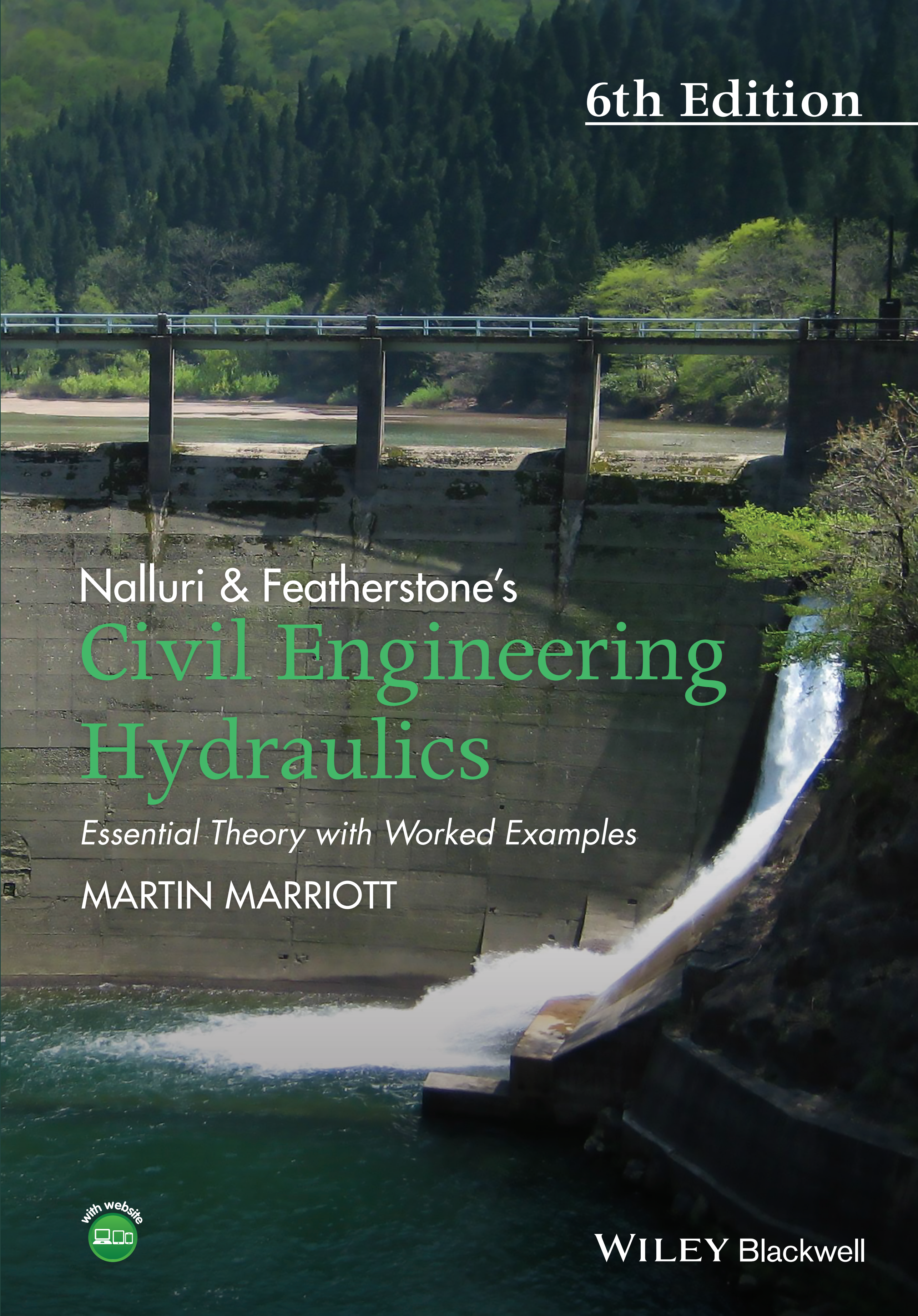

6th Edition

Nalluri & Featherstone's

Civil Engineering Hydraulics

Essential Theory with Worked Examples

MARTIN MARRIOTT

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Martin Marriott
University of East London

WILEY

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Preface to Sixth Edition

This book has regularly been on reading lists for hydraulics and water engineering modules for university civil engineering degree students. The concise summary of theory and the worked examples have been useful to me both as a practising engineer and as an academic.

The fifth edition aimed to retain all the good qualities of Nalluri and Featherstone's previous editions, with updating as necessary and with an additional chapter on environmental hydraulics and hydrology.

The latest sixth edition now adds a new chapter on coastal engineering prepared by my colleague Dr Ravindra Jayaratne based on original material and advice from Dr Dominic Hames of HR Wallingford. As before, each chapter contains theory sections, after which there are worked examples followed by a list of references and recommended reading. Then there are further problems as a useful resource for students to tackle. The numerical answers to these are at the back of the book, and solutions are available to download from the publisher's website: **http://www.wiley.com/go/Marriott.**

I am grateful to all those who have helped me in many ways, either through their advice in person or through their published work, and of course to the many students with whom I have enjoyed studying this material.

Martin Marriott
University of East London
2016

About the Author

This well-established text draws on Nalluri and Featherstone's extensive teaching experience at Newcastle University, including material provided by Professor J. Saldarriaga of the University of Los Andes, Colombia. The text has been updated and extended by Dr Martin Marriott with input from Dr Ravindra Jayaratne of the University of East London and Dr Dominic Hames of HR Wallingford.

Martin Marriott is a chartered civil engineer, with degrees from the Universities of Cambridge, Imperial College London and Hertfordshire. He has wide professional experience in the UK and overseas with major firms of consulting engineers, followed by many years of experience as a lecturer in higher education, currently at the University of East London.

Symbols

The following is a list of the main symbols used in this book (with their SI units, where appropriate). Various subscripts have also been used, for example to denote particular locations. Note that some symbols are inevitably used with different meanings in different contexts, and so a number of alternatives are listed below. Readers should be aware of this, and check the context for clarification.

a area (m^2); distance (m); acceleration (m/s^2)
b width (m); probability weighted moment of flows (m^3/s)
c wave celerity (m/s)
d diameter (m); water depth (m)
f force (N); function; silt factor; frequency
g gravitational acceleration ($\approx 9.81\ m/s^2$)
h height (m); pressure head difference (m); head loss (m)
i rank in descending order
j rank in ascending order
k radius of gyration (m); roughness height (m); constant; coefficient
m metacentric height (m); mass (kg)
n Manning's coefficient; exponent; number; wave steepness; group velocity parameter
p pressure (N/m^2)
q discharge per unit width (m^2/s)
r radius (m); discount rate
s relative density; distance (m); sinuosity; standard deviation of sample
t time (s); L-moment ratios
u velocity (m/s); parameter
v velocity (m/s)
w velocity (m/s)
x distance (m); variable
y distance (m); reduced variate; depth (m)
z elevation (m); vertical distance (m)

A area (m^2)
B width (m); centre of buoyancy; benefit
C constant; centre of pressure; coefficient; cost

D	diameter (m)
E	specifie energy (J/N = m); elastic modulus (N/m^2); wave energy (J/m^2)
F	force (N); head loss coefficient (s^2/m); annual probability of non-exceedance
Fr	Froude number
G	centroid
H	height (m); head (m); wave height (m)
I	second moment of area (m^4); inflow (m^3/s)
J	junction or node
K	bulk modulus of elasticity (N/m^2); coefficient; conveyance (m^3/s); circulation (m^2/s)
L	length (m); L-moment of flows (m^3/s); wavelength (m)
M	metacentre; mass (kg)
N	number; rotational speed (rev/min)
P	height of weir (m); wetted perimeter (m); power (W); annual exceedance probability; wave power (W/m)
Q	discharge (m^3/s)
R	resultant force (N); hydraulic radius (m); radius (m)
Re	Reynolds number
S	slope; energy gradient; storage volume (m^3); wave spectrum (m^{2}s)
T	thrust (N); time period (s); return period (years); surface width (m); thickness (m); wave period (s)
U	velocity (m/s)
V	volume (m^3); velocity (m/s)
W	weight (N); fall velocity (m/s)
We	Weber number
Z	elevation (m); section factor (m$^{5/2}$)
α	angular acceleration (rad/s^2); angle (rad); Coriolis coefficient; parameter
β	momentum correction factor (Boussinesq coefficient); parameter; slope
γ	specific weight (N/m^3)
δ	boundary layer thickness (m)
ζ	factor
η	efficiency; wave profile (m)
θ	angle (radian or degree); slope; wave direction
κ	constant
λ	Darcy–Weisbach friction factor; scale
μ	dynamic or absolute viscosity (Ns/m^2); ripple factor; mean
ν	kinematic viscosity (m^2/s)
ξ	spillway loss coefficient; displacement (m)
π	circle circumference-to-diameter ratio (≈ 3.142); Buckingham dimensionless group
ρ	mass density (kg/m^3)
σ	surface tension (N/m); safety factor
τ	shear stress (N/m^2)
ϕ	function; potential (m^2/s); transport parameter; angle of repose (degree)
ψ	stream function (m^2/s); flow parameter
ω	angular velocity (rad/s)
Δ	increment; submerged relative density

Chapter 1
Properties of Fluids

1.1 Introduction

A **fluid** is a substance which deforms continuously, or flows, when subjected to shear stresses. The term fluid embraces both gases and liquids; a given mass of liquid will occupy a definite volume whereas a gas will fill its container. Gases are readily compressible; the low compressibility, or elastic volumetric deformation, of liquids is generally neglected in computations except those relating to large depths in the oceans and in pressure transients in pipelines.

This text, however, deals exclusively with liquids and more particularly with Newtonian liquids (i.e. those having a linear relationship between shear stress and rate of deformation).

Typical values of different properties are quoted in the text as needed for the various worked examples. For more comprehensive details of physical properties, refer to tables such as Kaye and Laby (1995) or internet versions of such information.

1.2 Engineering units

The **metre–kilogram–second (mks) system** is the agreed version of the international system (SI) of units that is used in this text. The physical quantities in this text can be described by a set of three primary dimensions (units): mass (kg), length (m) and time (s). Further discussion is contained in Chapter 9 regarding dimensional analysis. The present chapter refers to the relevant units that will be used.

The unit of force is called newton (N) and 1 N is the force which accelerates a mass of 1 kg at a rate of 1 m/s^2 (1 N = 1 kg m/s^2).

The unit of work is called joule (J) and it is the energy needed to move a force of 1 N over a distance of 1 m. Power is the energy or work done per unit time and its unit is watt (W) (1 W = 1 J/s = 1 N m/s).

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1.3 Mass density and specific weight

Mass density (ρ) or **density** of a substance is defined as the mass of the substance per unit volume (kg/m^3) and is different from **specific weight** (γ), which is the force exerted by the earth's gravity (g) upon a unit volume of the substance ($\gamma = \rho g$: N/m^3). In a satellite where there is no gravity, an object has no specific weight but possesses the same density that it has on the earth.

1.4 Relative density

Relative density (s) of a substance is the ratio of its mass density to that of water at a standard temperature (4°C) and pressure (atmospheric) and is dimensionless.

For water, $\rho = 10^3$ kg/m^3, $\gamma = 10^3 \times 9.81 \simeq 10^4$ N/m^3 and $s = 1$.

1.5 Viscosity of fluids

Viscosity is that property of a fluid which by virtue of cohesion and interaction between fluid molecules offers resistance to shear deformation. Different fluids deform at different rates under the action of the same shear stress. Fluids with high viscosity such as syrup deform relatively more slowly than fluids with low viscosity such as water.

All fluids are viscous and 'Newtonian fluids' obey the linear relationship

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} \quad \text{(Newton's law of viscosity)} \qquad [1.1]$$

where τ is the shear stress (N/m^2), du/dy the velocity gradient or the rate of deformation (rad/s) and μ the coefficient of dynamic (or absolute) viscosity (N s/m^2 or kg/(m s)).

Kinematic viscosity (ν) is the ratio of dynamic viscosity to mass density expressed in metres squared per second.

Water is a Newtonian fluid having a dynamic viscosity of approximately 1.0×10^{-3} N s/m^2 and kinematic viscosity of 1.0×10^{-6} m^2/s at 20°C.

1.6 Compressibility and elasticity of fluids

All fluids are compressible under the application of an external force and when the force is removed they expand back to their original volume, exhibiting the property that stress is proportional to volumetric strain.

$$\text{The bulk modulus of elasticity}, K = \frac{\text{pressure change}}{\text{volumetric strain}} = -\frac{\mathrm{d}p}{(\mathrm{d}V/V)} \qquad [1.2]$$

The negative sign indicates that an increase in pressure causes a decrease in volume.

Water with a bulk modulus of 2.1×10^9 N/m^2 at 20°C is 100 times more compressible than steel, but it is ordinarily considered incompressible.

1.7 Vapour pressure of liquids

A liquid in a closed container is subjected to partial vapour pressure due to the escaping molecules from the surface; it reaches a stage of equilibrium when this pressure reaches saturated vapour pressure. Since this depends upon molecular activity, which is a function of temperature, the vapour pressure of a fluid also depends upon its temperature and increases with it. If the pressure above a liquid reaches the vapour pressure of the liquid, boiling occurs; for example, if the pressure is reduced sufficiently, boiling may occur at room temperature.

The saturated vapour pressure for water at 20°C is 2.45×10^3 N/m^2.

1.8 Surface tension and capillarity

Liquids possess the properties of cohesion and adhesion due to molecular attraction. Due to the property of cohesion, liquids can resist small tensile forces at the interface between the liquid and air, known as **surface tension** (σ: N/m). If the liquid molecules have greater adhesion than cohesion, then the liquid sticks to the surface of the container with which it is in contact, resulting in a capillary rise of the liquid surface; a predominating cohesion, in contrast, causes capillary depression. The surface tension of water is 73×10^{-3} N/m at 20°C.

The capillary rise or depression h of a liquid in a tube of diameter d can be written as

$$h = \frac{4\sigma \cos\theta}{\rho g d} \qquad [1.3]$$

where θ is the angle of contact between liquid and solid.

Surface tension increases the pressure within a droplet of liquid. The internal pressure p balancing the surface tensional force of a small spherical droplet of radius r is given by

$$p = \frac{2\sigma}{r} \qquad [1.4]$$

Worked examples

Example 1.1

The density of an oil at 20°C is 850 kg/m^3. Find its relative density and kinematic viscosity if the dynamic viscosity is 5×10^{-3} kg/(m s).

Solution:

$$\text{Relative density, } s = \frac{\rho \text{ of oil}}{\rho \text{ of water}} = \frac{850}{10^3} = 0.85$$

$$\text{Kinematic viscosity, } \nu = \frac{\mu}{\rho} = \frac{5 \times 10^{-3}}{850} = 5.88 \times 10^{-6}\ \text{m}^2/\text{s}$$

Example 1.2

If the velocity distribution of a viscous liquid ($\mu = 0.9$ N s/m^2) over a fixed boundary is given by $u = 0.68y - y^2$, in which u is the velocity (in metres per second) at a distance y (in metres) above the boundary surface, determine the shear stress at the surface and at $y = 0.34$ m.

Solution:

$$u = 0.68y - y^2$$
$$\Rightarrow \frac{du}{dy} = 0.68 - 2y$$

Hence, $(du/dy)_{y=0} = 0.68\ \text{s}^{-1}$ and $(du/dy)_{y=0.34\text{m}} = 0$.

$$\text{Dynamic viscosity of the fluid, } \mu = 0.9\ \text{N s/m}^2$$

From Equation 1.1,

$$\text{shear stress } (\tau)_{y=0} = 0.9 \times 0.68 = 0.612\ \text{N/m}^2$$

and at $y = 0.34$ m, $\tau = 0$.

Example 1.3

At a depth of 8.5 km in the ocean the pressure is 90 MN/m^2. The specific weight of the sea water at the surface is 10.2 kN/m^3 and its average bulk modulus is 2.4×10^6 kN/m^2. Determine (a) the change in specific volume, (b) the specific volume and (c) the specific weight of sea water at 8.5 km depth.

Solution:

$$\text{Change in pressure at a depth of 8.5 km, } dp = 90\ \text{MN/m}^2 = 9 \times 10^4\ \text{kN/m}^2$$

$$\text{Bulk modulus, } K = 2.4 \times 10^6\ \text{kN/m}^2$$

$$\text{From } K = -\frac{dp}{(dV/V)}$$

$$\frac{dV}{V} = \frac{-9 \times 10^4}{2.4 \times 10^6} = -3.75 \times 10^{-2}$$

Defining specific volume as $1/\gamma$ (m^3/kN), the specific volume of sea water at the surface = $1/10.2 = 9.8 \times 10^{-2}$ m^3/kN.

Change in specific volume between that at the surface and at 8.5 km depth, dV

$$= -3.75 \times 10^{-2} \times 9.8 \times 10^{-2}$$
$$= -36.75 \times 10^{-4}\ \text{m}^3/\text{kN}$$

$$\text{The specific volume of sea water at 8.5 km depth} = 9.8 \times 10^{-2} - 36.75 \times 10^{-4}$$
$$= 9.44 \times 10^{-2}\ \text{m}^3/\text{kN}$$

$$\text{The specific weight of sea water at 8.5 km depth} = \frac{1}{\text{specific volume}}$$
$$= \frac{1}{9.44 \times 10^{-2}}$$
$$= 10.6\ \text{kN/m}^3$$

References and recommended reading

Kaye, G. W. C. and Laby, T. H. (1995) *Tables of Physical and Chemical Constants*, 16th edn, Longman, London. http://www.kayelaby.npl.co.uk

Massey, B. S. and Ward-Smith, J. (2012) *Mechanics of Fluids*, 9th edn, Taylor & Francis, Abingdon, UK.

Problems

1. **(a)** Explain why the viscosity of a liquid decreases while that of a gas increases with an increase of temperature.

(b) The following data refer to a liquid under shearing action at a constant temperature. Determine its dynamic viscosity.

du/dy (s^{-1})	0	0.2	0.4	0.6	0.8
τ (N/m^2)	0	0	1.9	3.1	4.0

2. A 300 mm wide shaft sleeve moves along a 100 mm diameter shaft at a speed of 0.5 m/s under the application of a force of 250 N in the direction of its motion. If 1000 N of force is applied, what speed will the sleeve attain? Assume the temperature of the sleeve to be constant and determine the viscosity of the Newtonian fluid in the clearance between the shaft and its sleeve if the radial clearance is estimated to be 0.075 mm.

3. A shaft of 100 mm diameter rotates at 120 rad/s in a bearing 150 mm long. If the radial clearance is 0.2 mm and the absolute viscosity of the lubricant is 0.20 kg/(m s), find the power loss in the bearing.

4. A block of dimensions 300 mm × 300 mm × 300 mm and mass 30 kg slides down a plane inclined at 30° to the horizontal, on which there is a thin film of oil of viscosity 2.3×10^{-3} N s/m^2. Determine the speed of the block if the film thickness is estimated to be 0.03 mm.

5. Calculate the capillary effect (in millimetres) in a glass tube of 6 mm diameter when immersed in (i) water and (ii) mercury, both liquids being at 20°C. Assume σ to be 73×10^{-3} N/m for water and 0.5 N/m for mercury. The contact angles for water and mercury are 0 and 130°, respectively.

6. Calculate the internal pressure of a 25 mm diameter soap bubble if the tension in the soap film is 0.5 N/m.

Chapter 2
Fluid Statics

2.1 Introduction

Fluid statics is the study of pressures throughout a fluid at rest and the pressure forces on finite surfaces. Since the fluid is at rest there are no shear stresses in it. Hence the pressure p at a point on a plane surface (inside the fluid or on the boundaries of its container), defined as the limiting value of the ratio of normal force to surface area as the area approaches zero size, always acts normal to the surface and is measured in newtons per square metre (pascals, Pa) or in bars (1 bar = 10^5 N/m^2 or 10^5 Pa).

2.2 Pascal's law

Pascal's law states that the pressure at a point in a fluid at rest is the same in all directions. This means it is independent of the orientation of the surface around the point.

Consider a small triangular prism of unit length surrounding the point in a fluid at rest (Figure 2.1).

Since the body is in static equilibrium, we can write

$$p_1(\mathrm{AB} \times l) - p_3(\mathrm{BC} \times l)\cos\theta = 0 \qquad \text{(i)}$$

and

$$p_2(\mathrm{AC} \times l) - p_3(\mathrm{BC} \times l)\sin\theta - W = 0 \qquad \text{(ii)}$$

From Equation (i) $p_1 = p_3$, since $\cos\theta = \mathrm{AB/BC}$, and Equation (ii) gives $p_2 = p_3$, since $\sin\theta = \mathrm{AC/BC}$ and $W = 0$ as the prism shrinks to a point.

$$\Rightarrow p_1 = p_2 = p_3$$

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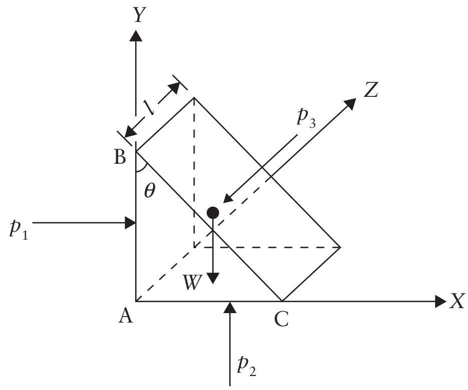

Figure 2.1 Pressure at a point.

2.3 Pressure variation with depth in a static incompressible fluid

Consider an elementary cylindrical volume of fluid (of length L and cross-sectional area dA) within the static fluid mass (Figure 2.2), p being the pressure at an elevation of y and dp being the pressure variation corresponding to an elevation variation of dy.

For equilibrium of the elementary volume,

$$p\,\mathrm{d}A - \rho g\,\mathrm{d}A\,L\sin\theta - (p + \mathrm{d}p)\,\mathrm{d}A = 0$$

or

$$\mathrm{d}p = -\rho g\,\mathrm{d}y \quad \left(\text{since}\sin\theta = \frac{\mathrm{d}y}{L}\right) \tag{2.1}$$

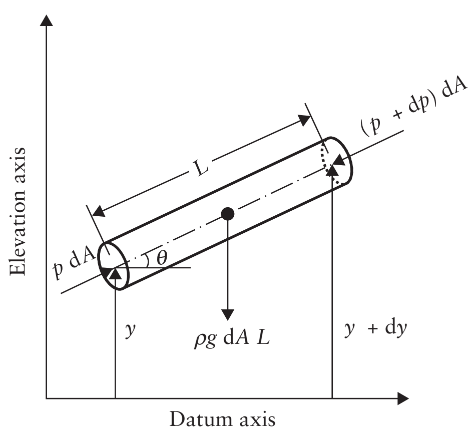

Figure 2.2 Pressure variation with elevation.

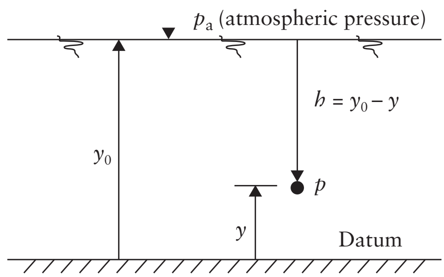

Figure 2.3 Pressure and pressure head at a point.

ρ being constant for incompressible fluids, we can write

$$\int \mathrm{d}p = -\rho g \int \mathrm{d}y$$

which gives

$$p = -\rho g y + C \tag{i}$$

When $y = y_0, p = p_a$, the atmospheric pressure (Figure 2.3).
From Equation (i),

$$\begin{aligned} p - p_a &= \rho g(y_0 - y) \\ &= \rho g h \end{aligned}$$

$$\text{or the pressure at a depth } h, p = p_a + \rho g h$$
$$= \rho g h \text{ above atmospheric pressure} \tag{2.2}$$

Note:

(a) If $p = \rho g h, h = p/\rho g$ and is known as the pressure head (in metres) of fluid of density ρ.

(b) Equation (i) can be written as $p/\rho g + y =$ constant, which shows that any increase in elevation is compensated by a corresponding decrease in pressure head. $(p/\rho g + y)$ is known as the piezometric head and such a variation is known as the hydrostatic pressure distribution.

If the static fluid is a compressible liquid, ρ is no longer constant and it is dependent on the degree of its compressibility. Equations 1.2 and 2.1 yield the relationship

$$\frac{1}{\rho} = \frac{1}{\rho_0} - \frac{gh}{K} \tag{2.3}$$

where ρ is the density at a depth h below the free surface at which its density is ρ_0.

2.4 Pressure measurement

The pressure at the earth's surface depends upon the air column above it. At sea level this atmospheric pressure is about 101 kN/m^2, equivalent to 10.3 m of water or 760 mm of mercury columns. A perfect vacuum is an empty space where the pressure is zero. **Gauge pressure** is the pressure measured above or below atmospheric pressure. The pressure

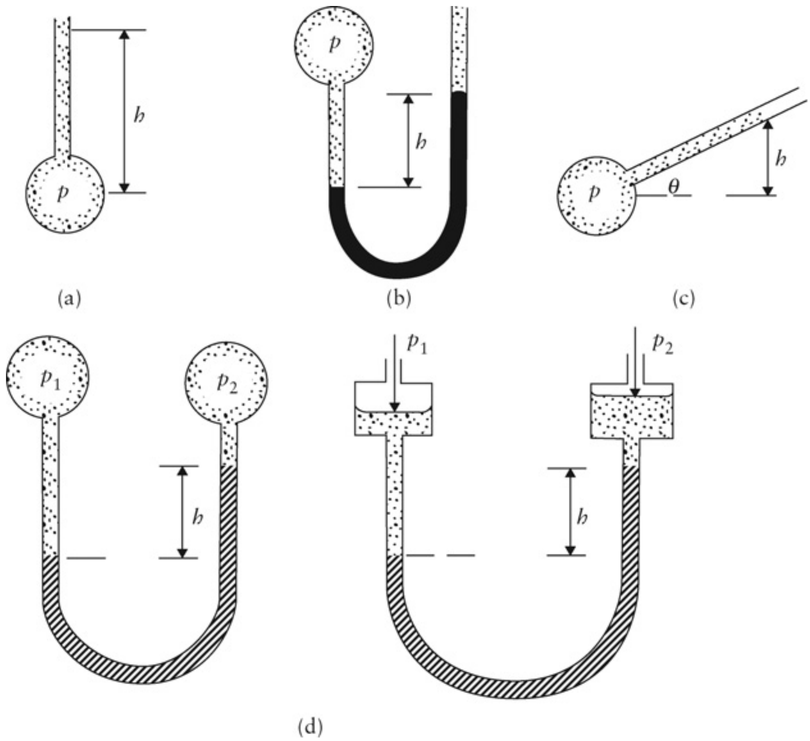

Figure 2.4 Pressure measurement devices: (a) piezometer, (b) U-tube, (c) inclined manometer and (d) differential manometers.

below atmospheric pressure is also called **negative or partial vacuum pressure. Absolute pressure** is the pressure measured above a perfect vacuum, the absolute zero.

(a) A simple vertical tube fixed to a system whose pressure is to be measured is called a piezometer (Figure 2.4a). The liquid rises to such a level that the liquid column's height balances the pressure inside.

(b) A bent tube in the form of a U, known as a U-tube manometer, is much more convenient than a simple piezometer. Heavy immiscible manometer liquids are used to measure large pressures, and lighter liquids to measure small pressures (Figure 2.4b).

(c) An inclined tube or U-tube (Figure 2.4c) is used as a pressure-measuring device when the pressures are very small. The accuracy of measurement is improved by providing suitable inclination.

(d) A differential manometer (Figure 2.4d) is essentially a U-tube manometer containing a single liquid capable of measuring large pressure differences between two systems. If the pressure difference is very small, the manometer may be modified by providing enlarged ends and two different liquids in the two limbs and is called a differential micromanometer.

If the density of water is ρ, a water column of height h produces a pressure $p = \rho g h$ and this can be expressed in terms of any other liquid column h_1 as $\rho_1 g h_1$, ρ_1 being its density:

$$\Rightarrow h \text{ in water column} = \left(\frac{\rho_1}{\rho}\right) h_1 = sh_1 \qquad [2.4]$$

where s is the relative density of the liquid.

For each one of the pressure measurement devices shown in Figure 2.4, an equation can be written using the principle of hydrostatic pressure distribution, expressing the pressures (in metres) of the water column (Equation 2.4) for convenience.

2.5 Hydrostatic thrust on plane surfaces

Let the plane surface be inclined at an angle θ to the free surface of water, as shown in Figure 2.5.

If the plane area A is assumed to consist of elemental areas dA, the elemental forces dF always normal to the surface area are parallel. Therefore the system is equivalent to one resultant force F, known as the **hydrostatic thrust**. Its point of application C, which would produce the same moment effects as the distributed thrust, is called the **centre of pressure**.

We can write

$$\begin{aligned} F = \int_A \mathrm{d}F = \int_A \rho g h \,\mathrm{d}A &= \rho g \sin\theta \int_A \mathrm{d}A\, x \\ &= \rho g \sin\theta\, A\bar{x} \\ &= \rho g \bar{h} A \qquad [2.5] \end{aligned}$$

where $\bar{h}$ is the vertical depth of the centroid G.

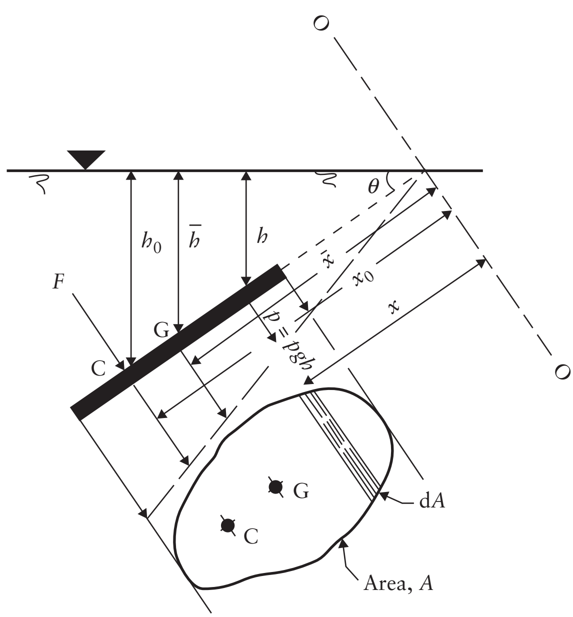

Figure 2.5 Hydrostatic thrust on a plane surface.

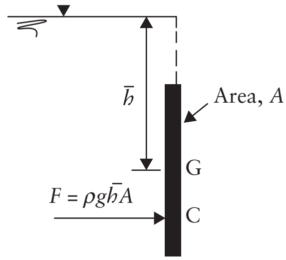

Figure 2.6 Vertical plane surface.

Taking moments of these forces about the axis O–O shown in Figure 2.5,

$$Fx_0 = \rho g \sin\theta \int_A \mathrm{d}A\, x^2$$

The distance to the centre of pressure, C, is therefore

$$\begin{aligned} x_0 &= \frac{\int_A \mathrm{d}A\, x^2}{\int_A \mathrm{d}A\, x} \\ &= \frac{\text{second moment of the area about axis O–O}}{\text{first moment of the area about axis O–O}} \\ &= \frac{I_0}{A\bar{x}} \end{aligned} \qquad [2.6]$$

But $I_0 = I_g + A\bar{x}^2$ (parallel-axis rule), where I_g is the second moment of area of the surface about an axis through its centroid and parallel to axis O–O.

$$\Rightarrow x_0 = \bar{x} + \frac{I_g}{A\bar{x}} \qquad [2.7]$$

which shows that the centre of pressure is always below the centroid of the area.

Depth of centre of pressure below free surface, $h_0 = x_0 \sin\theta$

$$\Rightarrow h_0 = \bar{h} + \frac{I_g \sin^2\theta}{A\bar{h}} \qquad [2.8]$$

For a vertical surface, $\theta = 90°$.

$$\Rightarrow h_0 = \bar{h} + \frac{I_g}{A\bar{h}} \qquad [2.9]$$

The distance between the centroid and centre of pressure is

$$\mathrm{GC} = \frac{I_g}{A\bar{h}} \quad \text{(see Figure 2.6)} \qquad [2.10]$$

The moment of F about the centroid is written as

$$\begin{aligned} F \times \mathrm{GC} &= \rho g \bar{h} A \times \frac{I_g}{A\bar{h}} \\ &= \rho g I_g \end{aligned}$$

which is independent of the depth of submergence.