Who Doesn't Love Puppies?!

2

Using Tree Diagrams

MATERIALS

None

Lesson Overview

Students use experimental data to create a probability model and then construct a second probability model using theoretical probabilities for comparison purposes. Tree diagrams are introduced as another method to illustrate all the possible outcomes in a sample space. Students then analyze a given tree diagram modeling the same situation and create a third probability model. In the second activity, a five-number spinner and a tree diagram are used to generate all possible outcomes to create a probability model and answer related questions. To demonstrate understanding, students create a tree diagram for all possible outcomes of correctly guessing the answers to a three-question true-or-false test. They then use the tree diagram to create a probability model and use the model to determine specified probabilities.

Grade 7
Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.
The student is expected to:

(A) represent sample spaces for simple and compound events using lists and tree diagrams.
(C) make predictions and determine solutions using experimental data for simple and compound events.
(D) make predictions and determine solutions using theoretical probability for simple and compound events.

ELPS

1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- Another method to determine the theoretical probability of an event is to construct a tree diagram.
- A *tree diagram* is a tree-shaped diagram that illustrates the possible outcomes of a given situation.
- A tree diagram shows how each possible outcome of an event affects the probabilities of the other events.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Three Puppies, Three Females

Students simulate the event of a litter of 3 puppies being all females using coin tosses. They construct a probability model using the results from the simulation.

Develop

Activity 2.1: Introduction to Tree Diagrams

Student construct a probability model to demonstrate the theoretical probability of the simulation conducted in the Getting Started. Tree diagrams are then introduced as another method for determining theoretical probability. Students analyze a tree diagram of the same situation to create a probability model that matches the probability model created from theoretical probabilities.

Day 2

Activity 2.2: Using Tree Diagrams To Determine Probabilities

Students consider the possible results from spinning a five-number spinner twice and computing the product of the two numbers. They construct a tree diagram to determine all the possible outcomes and list the product of each outcome. Using the tree diagram, students construct a probability model. Using the probability model, they calculate the probability of several events and answer related questions about complementary events.

Demonstrate

Talk the Talk: True-False Trees

Students create a tree diagram for all possible outcomes of correctly guessing the answers to a three-question true-or-false test. They then use the tree diagram to create a probability model, and they use the model to determine specified probabilities.

Getting Started: Three Puppies, Three Females

ENGAGE

Facilitation Notes

In this activity, coin tosses are used to simulate the birth of three females in a litter of three puppies. A probability model is constructed using the results from the simulation.

Ask a student to read the information. Complete Question 1 as a class, and discuss the different ways to simulate this event.

Differentiation strategy

There are many ways to simulate this event. You can allow each group to choose their own method or decide as a class to all use the same method. One way to simulate the event of a litter of puppies being comprised of three females is to use 3 coin flips; let heads represent a female, and let tails represent a male.

Have students work with a partner or in a group to complete Questions 2 through 5. Share responses as a class.

Questions to ask

- How did you represent the birth of a female?
- Is there a faster way to generate this experimental data?
- What is another way to generate this experimental data?
- Is there a difference between the outcome HTH, THH, and HHT? What is the difference?
- What do the outcomes HTH, THH, and HHT have in common?
- How does your probability model compare to your classmates' probability model?
- What do you notice about the sum of the probabilities for all possible events in each groups' model?
- Is this probability model experimental or theoretical? Explain.
- Is this probability model a uniform probability model or a non-uniform probability model? Explain.

Summary

A simulation, such as a coin toss, can be used as a probability model. This model can be used to calculate experimental probabilities.

DEVELOP

Activity 2.1

Introduction to Tree Diagrams

Facilitation Notes

Student construct a probability model to demonstrate the theoretical probability of the simulation conducted in the Getting Started. Tree diagrams are introduced as another method for determining theoretical probability. Students use the tree diagram to create a probability model that matches the probability model created from theoretical probabilities.

Ask a student to read the introduction aloud. Then, discuss the information as a class. Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

Questions to ask

- Are there more than 3 possible outcomes? Explain.
- How many outcomes are possible? What is the difference between the MFF outcome and the FFM outcome?
- How is the MFF outcome similar to the FFM outcome?
- Is this probability model experimental or theoretical? Explain.

Ask a student to read the information and definition aloud. Discuss the Worked Example as a class. Have students work with a partner or in a group to complete Questions 6 through 12. Share responses as a class.

Questions to ask

- Where is the outcome FFM in the tree diagram?
- Where is the outcome MFF in the tree diagram?
- Where is the outcome FMF in the tree diagram?
- Where is the outcome MFM in the tree diagram?
- Where is the outcome FMM in the tree diagram?
- What is an easy way to calculate the total number of outcomes in the tree diagram?
- If the dog had a litter of 4 puppies, how many rows would the tree diagram have?
- If the dog had a litter of 4 puppies, what would the fourth row of the tree diagram look like?
- If the dog had a litter of 4 puppies, what would be the total number of outcomes?
- Is the probability model resulting from the tree diagram based on experimental probabilities or theoretical probabilities?
- What is an advantage to using a tree diagram to determine probabilities?

- What is a disadvantage to using a tree diagram to determine probabilities?
- Is there a situation in which a tree diagram would prove difficult to create?
- What is an example of a situation in which a tree diagram would be difficult to create?

Summary

A tree diagram is visual representation used to organize outcomes that involve more than one event.

Activity 2.2

Using Tree Diagrams To Determine Probabilities

Facilitation Notes

In this activity, students consider a five-number spinner and the possible results if it is spun twice, and they calculate the product. Students create a tree diagram to determine all the possible outcomes. Using the tree diagram, they construct a probability model. Using the probability model, they calculate the probability of several events and answer related questions about complementary events.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- What information in the problem helped you determine the first branch of the tree diagram?
- What numbers appear on the first branch of the tree diagram?
- What information in the problem helped you determine how many branches should be in the tree diagram?
- How many branches are in the tree diagram?
- How does the tree diagram help you determine the number of possible outcomes?
- How many outcomes are possible?
- How do you know you have listed all of the possible outcomes?
- Which product(s) is/are least likely to occur? Explain.
- Which product(s) is/are most likely to occur? Explain.
- What products are less than 10?
- What products are multiples of 5?
- What products are not multiples of 5?

Have students work with a partner or in a group to complete Questions 4 through 7. Share responses as a class.

Questions to ask

- What is the complementary event associated with *P*(product is 60)?
- What is the complementary event associated with *P*(product is 16)?
- What is the complementary event associated with *P*(product is less than 10)?
- Is 10 a possible outcome?
- Are all outcomes either even or odd?
- Is there an outcome that is neither even nor odd?
- Why is the sum of the probabilities of two complementary events always equal to 1?

Summary

A tree diagram allows you to see all the possible outcomes of an event and calculate probability. The sum of the probabilities of two complementary events is equal to one.

DEMONSTRATE

Talk the Talk: True-False Trees

Facilitation Notes

In this activity, students create a tree diagram for all possible outcomes of correctly guessing the answers to a three-question true-or-false test. They then use the tree diagram to create a probability model and use the model to determine specified probabilities.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Do you think there is another way to model the problem situation that might be faster than a tree diagram?
- What are the advantages to using a tree diagram to organize all the possible outcomes?
- What are the disadvantages to using a tree diagram to organize all the possible outcomes?

Summary

Tree diagrams can be used to create a probability model. A probability model can be used to determine theoretical probabilities.

Who Doesn't Love Puppies?!

2

Using Tree Diagrams

WARM UP

Nora's treat bag has 5 chocolate bars, 7 peanut butter cups, and 8 sour gummies. If she selects 1 piece of candy at random, determine each probability.

1. *P*(chocolate bar)
2. *P*(peanut butter cups)
3. *P*(sour gummy)
4. *P*(not sour gummy)

LEARNING GOALS

- Develop a probability model and use it to determine probabilities.
- Construct a tree diagram to determine the theoretical probability of an event.
- Construct and interpret a non-uniform probability model.

KEY TERM

- tree diagram

Organized lists and arrays are two strategies for analyzing experiments that have a number of different outcomes. How can you use tree diagrams to display outcomes and their probability of occuring?

Warm Up Answers

1. $\frac{1}{4}$
2. $\frac{7}{20}$
3. $\frac{2}{5}$
4. $\frac{3}{5}$

LESSON 2: Who Doesn't Love Puppies?! • 1103

Answers

1. Sample answer.
   You can use 3 coin flips to simulate the event of a litter of puppies being 3 females. Let heads represent a female, and let tails represent a male.
2. Sample answers.

Trial	Results
1	HTH
2	TTH
3	TTT
4	HHT
5	TTT
6	THT
7	TTH
8	HTH
9	HTH
10	TTH
11	THH
12	HTT
13	THT
14	HHH
15	HTT
16	HTT
17	TTH
18	THT
19	HTH
20	HTT
21	HTH
22	HHT
23	TTT
24	HHT
25	THH

Getting Started

Three Puppies, Three Females

What is the probability that if a dog has a litter of 3 puppies, those 3 puppies are females? Let's say that the theoretical probability of a female being born is equal to the theoretical probability of a male being born, which is $\frac{1}{2}$.

Let's simulate the event of a litter of 3 puppies being 3 females.

1. **Choose an appropriate model to simulate the probability of a litter of three puppies being all females. Explain how you will represent females and males in your model.**

2. **Conduct 25 trials of the simulation. Record the results in the table shown.**

Trial	Results	Trial	Results	Trial	Results
1		10		19	
2		11		20	
3		12		21	
4		13		22	
5		14		23	
6		15		24	
7		16		25	
8		17			
9		18			

3. List all possible outcomes for the number of females among a litter of 3 puppies.

4. Use the results from your simulation to construct a probability model.

Outcome	0 females	1 female	2 females	3 females
Probability				

5. What is the experimental probability that a litter of 3 puppies is all females according to your probability model?

Answers

3. {0, 1, 2, 3}
4. See sample table below.
5. Sample answer. According to the model, the probability is $\frac{1}{25}$.

4.

Outcome	0 females	1 female	2 females	3 females
Probability	$\frac{3}{25}$	$\frac{10}{25}$	$\frac{11}{25}$	$\frac{1}{25}$

Answers

1. Germaine is correct.

Karl is not correct. There are more than 3 total outcomes. For example, there could be 2 females and 1 male, but the first and last puppies could be females and the middle puppy a male, or the first 2 puppies could be females and the last puppy a male.

ACTIVITY 2.1

Introduction to Tree Diagrams

In the previous simulation, your probability model was based on experimental probabilities. In some cases, this is the only method of constructing a probability model. For example, in an earlier lesson, you determined the experimental probabilities of a cup landing on its top, bottom, or side when it was tossed. It would be difficult or impossible to determine the theoretical probabilities for the cup toss. However, it is possible to determine the theoretical probability for a litter of 3 puppies being 3 females.

One method to calculate the theoretical probability for a litter of puppies being 3 females is to list all of the possible outcomes for a litter of 3 puppies. You can then determine how many of those outcomes include 3 females.

1. Karl says, "I think that the probability of a litter of puppies being 3 females is 1 out of 3 because there is only one outcome that has all three puppies being female. There are only two other outcomes."

Germaine says, "I don't think that's correct. I think the probability is much lower since there are many combinations of males and females in a litter of three puppies."

Who's correct? Explain your reasoning.

NOTES

2. List all of the possible outcomes for having 3 puppies in a litter, using F to represent females and M to represent males.

3. What does the outcome MFF represent?

4. Complete the probability model using all possible outcomes.

Outcome	0 females	1 female	2 females	3 females
Probability				

5. What is the theoretical probability that a litter of 3 puppies is comprised of 3 females?

Answers

2. (FFF), (FFM), (FMF), (MFF), (FMM), (MFM), (MMF), (MMM)
3. MFF represents the litter being a male as the first puppy, a female as the second puppy, and a female as the third puppy.
4. See table below.
5. $\frac{1}{8}$

4.

Outcome	0 females	1 female	2 females	3 females
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Answers

6. The tree diagram would need to be extended to include a fourth row.
7. Both show the same possible outcomes.
8. See tree diagram below.

A tree diagram has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

Another method of determining the theoretical probability of an event is to construct a *tree diagram*. A **tree diagram** illustrates the possible outcomes of a given situation. Tree diagrams can be constructed vertically or horizontally.

You can construct a tree diagram to show all the possible outcomes for a litter of 3 puppies.

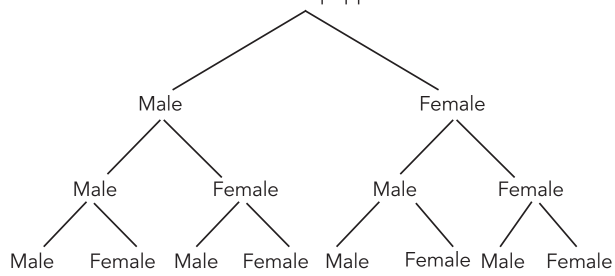
WORKED EXAMPLE

What are the possible outcomes for a litter of 3 puppies?

List the possible outcomes for the 1st puppy.

List the possible outcomes for the 2nd puppy.

List the possible outcomes for the 3rd puppy.



6. How would this tree diagram change if you were trying to determine the possible outcomes for a litter of 4 puppies?

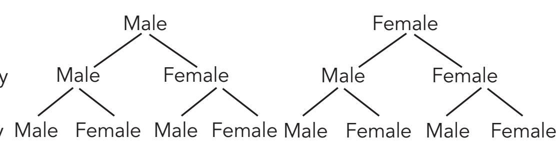
7. How does the tree diagram in the Worked Example compare to the list you made in Question 2?

8. Circle the outcome(s) of a litter of three puppies that are all females on the tree diagrams shown.

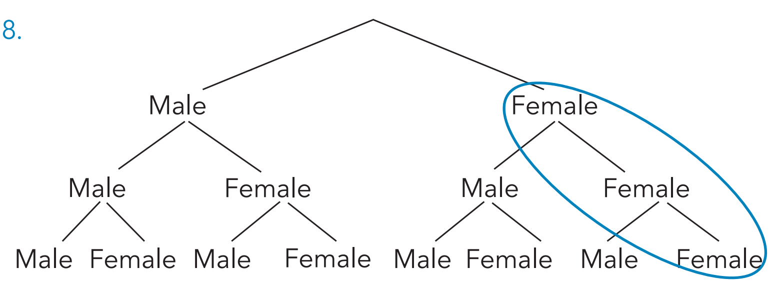
1st puppy

2nd puppy

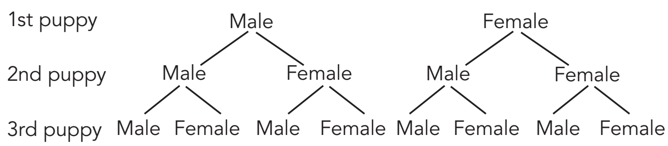
3rd puppy



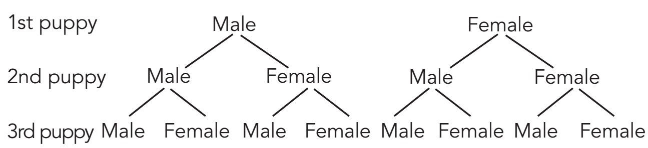
8.



9. **Circle the outcome(s) of a litter of three puppies in which two of the puppies are females in the tree diagrams shown.**



10. **Circle the outcome MMF in the tree diagrams shown.**



11. **Complete the probability model shown with the information from the tree diagrams.**

Outcome	0 females	1 female	2 females	3 females
Probability				

12. **Is there a difference in the theoretical probability of each outcome between the list of outcomes you wrote and the tree diagrams you analyzed?**

Answers

9. See tree diagram below.

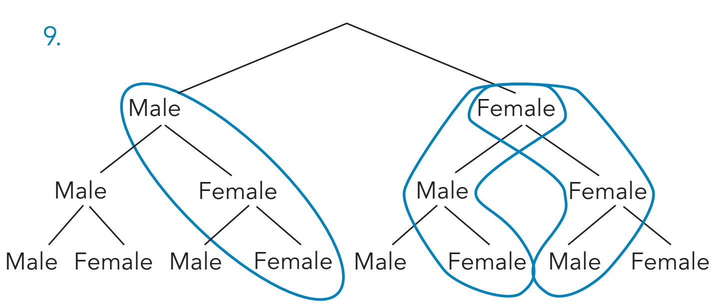
10. See tree diagram below.

11.

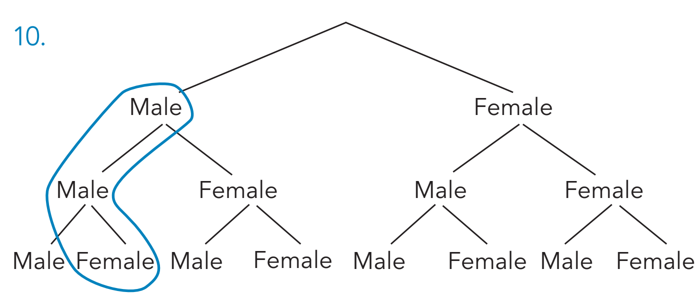
outcome	probability
0 females	$\frac{1}{8}$
1 female	$\frac{3}{8}$
2 females	$\frac{3}{8}$
3 females	$\frac{1}{8}$

12. No. The probability of each outcome is the same whether I create a list of all outcomes or use a tree diagram.

9.

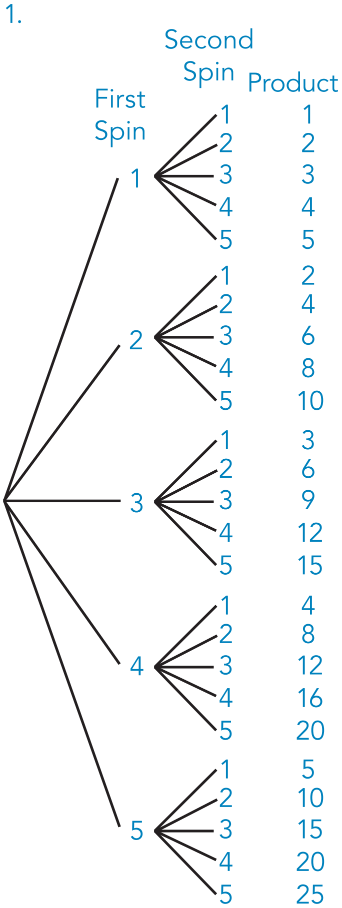


10.



Answers

1.



2. See tables below.

3a. $\frac{2}{25}$

3b. $\frac{15}{25}$

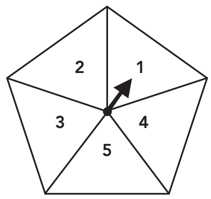
3c. $\frac{9}{25}$

3d. $\frac{16}{25}$

ACTIVITY 2.2

Using Tree Diagrams to Determine Probabilities

The five-number spinner is spun twice and a product is calculated.



1. Construct a tree diagram to determine all the possible outcomes. Then, list the product at the end of each branch of the tree.

Product	Probability
1	
2	
3	
4	
5	
6	
8	
9	
10	
12	
15	
16	
20	
25	

2. Construct a probability model for spinning the spinner twice and recording the product.

3. Use the probability model you created to calculate the probability for each event shown.

a. ***P*(10)**

b. ***P*(less than 10)**

c. ***P*(multiple of 5)**

d. ***P*(not a multiple of 5)**

2.

Product	1	2	3	4	5	6	8
Probability	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{2}{25}$	$\frac{3}{25}$	$\frac{2}{25}$	$\frac{2}{25}$	$\frac{2}{25}$

Product	9	10	12	15	16	20	25
Probability	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{2}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$

4. **Which events from Question 3 represent complementary events? Explain your reasoning.**

5. **Betina says that the product being less than 10 and the product being more than 10 are complementary events. Davika disagrees. Who is correct? Explain your reasoning.**

6. **What event is complementary to the event that the product is an even number? Determine the probability of both events.**

7. **What is the sum of the probabilities of two complementary events? Explain why your answer makes sense.**

Answers

4. The events of the product being a multiple of 5 and the product not being a multiple of 5 represent complementary events. Complementary events are events that consist of a desired outcome and the remaining events that consist of all the undesired outcomes. Together, they include every possible outcome in the sample space.
5. Betina is not correct. The product being less than 10 and the product being more than 10 do not include the entire sample space—the product being equivalent to 10 is not included with either event.
6. The product being an odd number would be complementary to the event that the product is an even number. *P*(product is an even number) is $\frac{16}{25}$, and *P*(product is an odd number) is $\frac{9}{25}$.
7. The sum of the probabilities of two complementary events is 1. This makes sense because complementary events together include the entire set of outcomes for an experiment.

LESSON 2: Who Doesn't Love Puppies?! • 1111

Answers

1. See tree diagram below.

2.

Outcome	Probability
3 correct	$\frac{1}{8}$
2 correct	$\frac{3}{8}$
1 correct	$\frac{3}{8}$
0 correct	$\frac{1}{8}$

3a. $\frac{1}{8}$

3b. $\frac{6}{8}$

3c. $\frac{1}{8}$

NOTES

TALK the TALK

True-False Trees

1. **Construct a tree diagram to determine all the possible outcomes for correctly guessing the answers to a three-question true-or-false quiz.**

2. **Construct a probability model for the tree diagram you completed in Question 1.**

3. **Calculate the probabilities:**

   a. ***P*(3 questions correct) =**

   b. ***P*(1 or 2 questions correct) =**

   c. ***P*(0 questions correct) =**

1.